1. CONDITIONAL TERM STRUCTURE OF INFLATION FORECAST UNCERTAINTY: THE COPULA APPROACH

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Abstract

The paper introduces the concept of conditional inflation forecast uncertainty. It is proposed that the joint and conditional distributions of the bivariate forecast uncertainty can be derived from the estimation of the unconditional distributions of these uncertainties and applying appropriate copula function. Empirical results have been obtained for Canada and the US. Term structure has been evaluated in the form of unconditional and conditional probabilities of hitting the inflation range of ±1% around the Canadian inflation target. The paper suggests a new measure of inflation forecast uncertainty that accounts for possible inter-country dependence. It is shown that the evaluation of targeting precision can be effectively improved with the use of ex-ante formulated conditional and unconditional probabilities of inflation being within the pre-defined band around the target.

Keywords: macroeconomic forecasting, inflation, uncertainty, non-normality, density forecasting, forecast term structure, copula modelling

JEL Classification: C53, E37, E52

1. Introduction

Stimulated by the current uncertain economic climate, there has been an increasing interest in the measurement and evaluation of macroeconomic uncertainty. The research has predominantly focused on the development of the univariate conditional measures of uncertainty, describing it either for particular macroeconomic indicators (usually inflation or output, see, *e.g.* Clements, 2014; Lahiri and Sheng, 2010; Lahiri, Peng and Sheng, 2014; Rossi and Sekhposyan, 2015; Rossi, Sekhposyan and Soupre, 2016 and others), or the aggregated macroeconomic, policy or behavioural uncertainty (Jurado, Ludvigson and Ng, 2015; Tuckett *et al.*, 2014; Baker, Bloom and Davis, 2016; Caldara and Iacoviello, 2018). These measures are usually significantly correlated among themselves, especially the indicators' measures and the aggregated measures, as the former are often incorporated

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within the latter. However, this correlation is, in some cases, disappearing; particularly, inflation and macroeconomic uncertainties become, on the surface, unrelated.

This paper claims that such lack of correlation might result from interrelations between inflation uncertainty for different countries. Section 2 motivates the research by presenting the rather puzzling result of such lack of correlation for Canada and its existence for a number of other countries. It is claimed that this was the result of conditioning inflation uncertainty in Canada on that in the US. Section 3 introduces measures and indicators of the bivariate, unconditional and conditional uncertainty. Section 4 gives the results of the estimation of the univariate (unconditional) uncertainties. Section 5 discusses the main results for Canada and shows that the probabilities of inflation in Canada being within the ±1% band around the target increases, especially for short forecast horizons, if conditioned on the US inflation being within similar bands. It shows that the evaluation of targeting precision can be effectively improved with the use of *ex-ante* formulated conditional and unconditional probabilities of inflation being within the pre-defined band around the target. It also suggests a new measure of inflation uncertainty that is less affected by external inflation than the measures based solely on forecast errors. Section 6 concludes.

2. Motivation: What Happened to Correlation between the Uncertainties?

The motivation for this research has been provided by the puzzling results of correlations between a rudimentary measure of inflation forecast uncertainty and the economic policy uncertainty. Inflation uncertainty is evaluated simply by the squares of forecast errors made from a univariate ARMA-GARCH model (see, *e.g.* Clements, 2014; Charemza, Díaz and Makarova, *forthcoming*). Table 1 contains Spearman's rank correlation coefficients of the logarithms of such squares of forecast errors for the forecast horizons from 1 to 12 months with the logarithms of economic policy uncertainty index (EPU), described by Baker, Bloom and Davis (2016) and available at http://www.policyuncertainty.com/ for selected countries. The EPU is a three-component index, based on: (a) the frequency of the use of word 'uncertainty' in leading newspapers, (b) tax code provisions and (c) disagreement between the forecasters (the so-called uncertainty by disagreement).[4] For the US, we have additionally included Spearman's rank correlation coefficients of the forecast errors with the Jurado, Ludvigson and Ng (2015) measure of macroeconomic uncertainty, denoted as JLN, with data described in Jurado, Ludvigson and Ng, (2014). The period for which the correlations are computed is from January 1997 until December 2012, where the last data on the JLN index is available. *P*-values of the correlation coefficients have been computed by simple bootstrap. They are not reported here, but the correlation coefficients that are not significant at 10% level are boldfaced.

Table 1 indicates that, except for Canada, there is a significant positive correlation between the squares of forecast errors and uncertainty measures for most forecasts horizons. Such correlation is in fact expected, as inflation forecast errors constitute a substantial component of macroeconomic uncertainty. However, for Canada, the correlation is predominantly insignificant. Closer inspection of data suggests that such breakdown in correlation was mainly caused by an unpredictable (by a univariate autoregressive model) fall in inflation in the first half of 1990's, where an earlier inflation drop in US preceded the decline in Canadian inflation and, therefore, foreseen by the Canadian media. As media information constitutes

[4] *For some countries only the first two components are applied.*

a relevant component of the political uncertainty, it affected the EPU earlier, then the changes in inflation happen.

Table 1

Spearman's Rank Correlation between Uncertainty Measures and Squares of Inflation Forecast Errors

	EPU							
	Canada	France	India	Italy	Spain	UK	US	US_JLN
1	**0.05**	**0.02**	0.10	0.13	0.23	0.31	0.09	0.34
2	**-0.03**	0.14	**0.10**	0.13	0.25	0.30	0.17	0.37
3	0.01	0.10	0.15	0.17	0.39	0.42	0.22	0.43
4	**0.06**	0.10	0.21	0.12	0.39	0.47	0.25	0.45
5	**0.09**	0.17	0.28	0.11	0.35	0.46	0.26	0.46
6	0.06	0.21	0.30	0.15	0.28	0.47	0.25	0.47
7	**0.09**	0.19	0.26	0.20	0.20	0.53	0.23	0.48
8	**0.14**	0.15	0.26	0.18	0.14	0.53	0.27	0.48
9	**0.17**	0.15	0.27	0.20	0.15	0.49	0.30	0.49
10	**0.16**	0.18	0.34	0.23	0.18	0.42	0.32	0.49
11	**0.17**	0.22	0.22	0.23	0.22	0.39	0.33	0.49
12	0.20	0.24	0.25	0.20	0.22	0.36	0.33	0.47

Ad-hoc reflection is that there might be an influence of the US inflation uncertainty on that of Canada. If the US inflation uncertainty affects, possibly with some lag, the Canadian uncertainty, a natural way to proceed would be to model the Canadian inflation jointly with the US inflation and analyse the Canadian inflation forecast uncertainty conditionally on that of the US.

3. Measuring the Dependence between Uncertainties

We traditionally define the observations on the *ex-post* forecast uncertainty for the forecast horizon *h* made at time $t-h$ as the rolling sequence of pseudo-out-of-sample forecast errors (see, *e.g.,* Stock and Watson, 2007). These forecasts are usually obtained from a time series econometric model and possibly adjusted for variance predictability. Under the assumptions of stationarity and ergodicity of these errors, we assume that they stand for realisations of a random variable, denoted by $U^{i}_{t,h}$, where *i* represents the *i*-th country.

We consider the bivariate *ex-post* forecast uncertainty for countries 1 and 2, $U_{t,h}=\left(U^{(1)}_{t,h},U^{(2)}_{t,h}\right)'$ given by:

$$U_{t,h}=\Sigma^{1/2}_{t,h}\Sigma^{-1/2}_{t|t-h}(\pi_t-\pi_{t|t-h}), \quad (1)$$

where: π_t is the bivariate vector containing the inflation in both countries in period *t*, $\pi_{t|t-h}$ is the vector containing the corresponding forecasts made at time *t-h* for the period *t*, $\Sigma_{t,h}$

is the unconditional covariance matrix of the *h* step ahead forecast errors at time *t* and $\Sigma_{t|t-h}$ is the conditional covariance matrix made at time *t-h* for time *t*. The variable $U_{t,h}$ is, then, net of all information available at the time of making the forecast regarding its first two moments. The bivariate density of $U_{t,h}$ is denoted by $D(0,\Sigma_{t,h})$. The unconditional distributions of $U_{t,h}^{(1)}$ and $U_{t,h}^{(2)}$ can be approximated by a variety of statistical distributions.

Unfortunately, the analytical forms of the bivariate distributions mentioned above might not be of much use here (even if they were known). Firstly, the dependence between forecast uncertainties might be different for lower and upper tails of their distributions and, for the policy analysis, asymmetric dependences of macroeconomic indicators might be of particular interest.

Secondly, due to the different monetary policies pursued by countries 1 and 2, types of the unconditional distributions might be different. For instance, country 1, which implements inflation targeting successfully, might have the distribution of inflation forecast errors well described by the WSN distribution, while country 2, which pursues a different policy, might have the empirical distribution of forecast errors better described by the TPN distribution.

In the light of these difficulties, we propose to evaluate the bivariate density of $U_{t,h}$ defined by (1) by approximating the unconditional densities using a univariate parametric density and then modelling the dependency using copulas. Let F_1 and F_2 be the unconditional cumulative distribution functions (*cdf*'s) of the uncertainties in both countries and f_1 and f_2 the corresponding probability density functions (*pdf*'s). We can obtain the joint *cdf* as

$$F_{12}\left(x_1,x_2\right)=C\left[u_1,u_2;\theta\right], \qquad (2)$$

where: $u_1=F_1\left(x_1\right)$, $u_2=F_2\left(x_2\right)$ with $x_1,x_2\in\mathrm{R}$ and $C:[0,1]^2\rightarrow[0,1]$ is a copula function which depends on parameter θ. Sklar's (1959) Theorem shows that if both unconditional *cdf*'s are continuous then, the copula is unique, so that $C(\cdot,\cdot)$ can be considered a *cdf* itself (we limit our interest here to one-parameter copulas). Also, if the copula is twice differentiable, we can define $c\left(u_1,u_2\mid\theta\right)=\partial^2C(u_1,u_2\mid\theta)/\partial u_1\partial u_2$ as the density function of the copula and, differentiating (2), we can express the joint density of (1) as

$$f_{12}(x_1,x_2)=c\left(u_1,u_2;\theta\right)\times f_1(x_1)\times f_2(x_2). \qquad (3)$$

Although the copula parameter θ can be estimated jointly with the parameters of the unconditional distributions by the maximum likelihood directly from (3), this can be numerically awkward if the unconditional distributions are difficult to estimate. Because of that, we use the *Inference Function for Margins* (*IFM*) approach described in Joe and Xu (1996). This is a two-steps estimation method which consists of:

1. estimating the parameters of the density functions of the unconditional distributions;

2. estimating the copula parameter by the maximum likelihood by plugging in the probability integral transforms (*pit*'s) of the marginals into the copula density (3). For the details of the algorithms see Durrelman, Nikeghbali and Roncalli (2000).

Finally, developing from the joint density of uncertainties (3), we can evaluate the density of inflation uncertainty in country 1 conditional on inflation in country 2 being in a certain range [a, b] around its point forecast as

$$f_{1|2}\left(x_1|a \le x_2 \le b\right) = \frac{\int_a^b f_{12}(x_1, x_2)dx_2}{\int_a^b f_2(x_2)dx_2} \quad . \tag{4}$$

Knowledge of (4) can be of relevant practical importance. In particular, policymakers in country 1 can assess the probabilities related to changes in monetary policy in country 2, for instance, the probability of hitting the inflation target band. More generally, they can evaluate the conditional term structure of inflation, which is changes in uncertainty with the changes in the forecast horizon (see Patton and Timmermann, 2011).

4. Estimating Univariate Forecast Uncertainties

Motivated by the puzzling lack of correlation between inflationary forecast errors and the EPU index for Canada, discussed in Section 2, we focus on the interrelations between the Canadian and US forecast uncertainties. The raw data we used are monthly data on annual CPI inflation in Canada and US from January 1985 until October 2014. As the Canadian inflation targeting is often discussed in terms of the core rather than headline inflation, we have also applied data on the core inflation for Canada[5]. Inflation in both countries has been found to be *I*(1); therefore, the model has been estimated in first differences, using 358 observations in total. The first recursion is made with 80 observations, which gives 278 one step ahead forecast errors, 257 two-step ahead errors, etc. In each recursion, for the time period until $t-h$, in order to account for second order predictability, the two-equation VAR-BEKK-GARCH(1,1) model for the Canadian and US inflation with seasonal dummies in its deterministic part has been estimated (for the discussion of the assumptions and properties of the BEKK-GARCH model and its comparison with other multivariate GARCH models see, *e.g.,* Silvennoinen and Teräsvirta, 2009). The autoregressive order of the model had been chosen as the minimum for which the residuals' autocorrelation is not significant at 5% significance level. In order to avoid spurious dependence between the corresponding *h*-step-ahead forecast errors for $h>1$, forecasts have been made from the moving average rather than autoregressive form of the model (see, *e.g.,* Lütkepohl, 2007, p. 94). Forecasting gives *h*-step ahead forecast errors $e_{t|t-h}$, up to *h*=24 months. The conditional and unconditional

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[5] *Data on US CPI are from the Bureau of Labor Statistics at http://www.bls.gov/cpi/ . Canadian price data are from CANSIM (http://www5.statcan.gc.ca/cansim/home-accueil?lang=eng). Data for core inflation for Canada are under header CANSIM-V41693242.*

variance-covariance matrices of $e_{t|t-h}$, denoted in (1) as $\Sigma_{t|t-h}$ and $\Sigma_{t,h}$, have been estimated using variance-covariance matrices obtained for the estimated VAR-BEKK-GARCH(1,1) model. Then, using a rolling window of the length of 120, we estimated 158 distributions of one-step-ahead forecasts for both countries, 157 of two-step-ahead forecasts and so on.

As the first step of the IFM estimation method is to evaluate the parameters of the unconditional distributions, we start with choosing the most appropriate distribution of the marginals. As this is somewhat arbitrary, we have decided to choose from two distributions used for modelling forecast uncertainties, namely the two-piece normal (TPN; see Tay and Wallis, 2000; Wallis, 2004) and the weighted skew-normal (WSN, see Charemza, Díaz and Makarova, *forthcoming*). Parameters of these distributions can be interpreted in the context of policy effects. The TPN has the density function with three parameters and is defined by

$$f_{TPN}(t;\sigma_1,\sigma_2,\mu)=\begin{cases} A\exp\left\{-(t-\mu)^2/2\sigma_1^2\right\} & \text{if } t\le\mu \\ A\exp\left\{-(t-\mu)^2/2\sigma_2^2\right\} & \text{if } t>\mu \end{cases},$$

where: $A=\left(\sqrt{2\pi}(\sigma_1+\sigma_2)/2\right)^{-1}$. If $\sigma_1^2=\sigma_2^2$ it becomes normal and the deviations from normality (that is the differences between the estimates of σ_1 and σ_2) are interpreted as the effects of the balance of risks given by over-and underestimated forecasts (see Wallis, 2004). WSN is the 5-parameters' distribution, with the density function given, after normalisation $U^*=U/\sigma$, where σ is the standard deviation of U, as:

$$f_{WSN_1}(t;\alpha,\beta,m,k,\rho)=\frac{1}{\sqrt{A_\alpha}}\varphi\left(\frac{t}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha t-mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right)+\frac{1}{\sqrt{A_\beta}}\varphi\left(\frac{t}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta t+kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right),$$
$$+\varphi(t)\cdot\left[\Phi\left(\frac{m-\rho t}{\sqrt{1-\rho^2}}\right)-\Phi\left(\frac{k-\rho t}{\sqrt{1-\rho^2}}\right)\right]$$

where φ and Φ denote the density and cumulative distribution functions of the standard normal distribution, respectively, $A_\tau=1+2\tau\rho+\tau^2$, and $B_\tau=\tau+\rho$. If $\alpha=\beta=0$, WSN reduces to normal distribution. In the general case, parameters $\alpha<0$ and $\beta<0$ can be interpreted as the effects of the anti- and pro-inflationary policy respectively in reducing inflation uncertainty, m and k represent the tolerance level to the nuisance (not strong enough) forecast signals coming from outside of the model and $\rho\in(0,1)$ describes the degree of accuracy of these forecast signals (see Charemza, Díaz and Makarova, *forthcoming*).

It is shown that the maximum likelihood estimation of skew-normal distributions can be subject to bias and convergence problems (see, *e.g.,* Pewsey, 2000, Monti, 2003). Therefore, the estimation procedure applied here is the Simulated Minimum Distance Estimator (SMDE) method of Charemza *et al.* (2012). The SMDE is defined as

$$\hat{\omega}_n^{SMDE} = \arg\min_{\omega\in\Omega}\left\{\xi\left\{HD(d_n\ ,\ f_{r,\omega}\right\}_{r=1}^{R}\right\} \quad ,$$

where $\omega \in \Omega \subset R^k$, $f_{r,\omega}$ is the Monte Carlo approximation of the theoretical probabilities of the estimated distribution obtained from *R* replications for each combination of parameters within the admissible area, d_n denotes the density of an empirical sample of size *n*, *HD* is the distance measure and ξ is an aggregation operator. This method, albeit relatively slow and not very precise (as it relies on the accuracy of the grid search algorithm applied), does not, however, suffer from convergence problems. The distance measure chosen here is the Hellinger distance (see, *e.g.,* Basu, Shioya and Park*,* 2011) which is known to be robust to outliers. To make results comparable, three parameters have been estimated for each distribution: σ_1^2 , σ_2^2 and μ for the TPN and α, β and σ, with three remaining parameters fixed as ρ = 0.75 and $m = -k = 1$.

Detailed estimation results, for all forecast horizons and all rolling windows, are available at *http://pramu.ac.uk.* Selection of unconditional distributions has been made using the forecast accuracy tests, also available at *http://pramu.ac.uk*. The tests applied are: (1) the Cramer-von Mises test of uniformity of the probability integral transforms (*pit*'s), Jarque-Bera test of normality of *pit*'s transformed to normality (see Berkowitz, 2001) and, (3) the Amisano and Giacomini (2007) test for direct comparison of the distributions. Results of all these tests almost universally support the superiority of WSN over TPN for both Canada and US. Consequently, we base a further investigation on using WSN as the unconditional distributions for both countries.

5. Copula Estimation and Conditional Forecasting

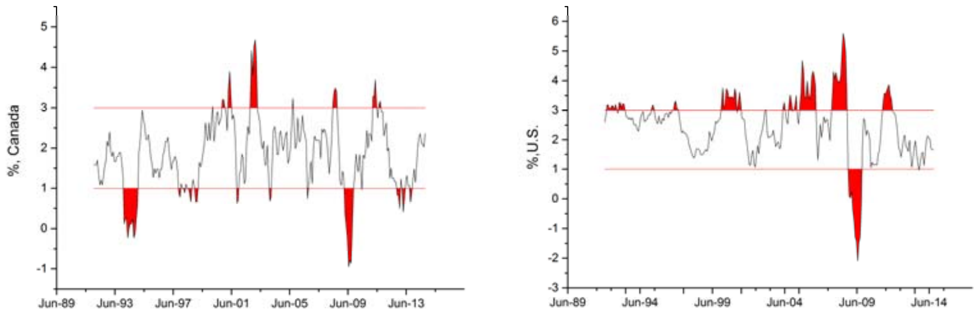
Once the unconditional distributions of uncertainties are decided, we model the joint density as in (3). We experimented with a number of different copula functions, and we have finally decided to use Frank's copula, as it was capable of modelling strong asymmetric dependence between non-normal skewed distributions, without favouring neither the upper nor the lower tail (for some discussion of the properties of Frank's copula see, *e.g.,* Assunção, 2004, Lin and Wu, 2015). The expressions of this copula and its density are, respectively

$$C\left(u_1,u_2;\theta\right) = -\theta^{-1}\log([\eta-(1-e^{-\theta u_1})(1-e^{-\theta u_2})]/\eta)\ ,$$
$$c\left(u_1,u_2;\theta\right) = \theta\eta e^{-\theta(u_1+u_2)}/[\eta-(1-e^{-\theta u_1})(1-e^{-\theta u_2})]^2 \quad (5)$$

where the copula parameter $\theta \in [0,+\infty)$ and $\eta = \theta - 1$. Following the IFM procedure, we estimate the copula parameter by maximising $\sum_{t=1}^{T}\log c\left(u_{1t},u_{2t};\theta\right)$. The conditional density can be then evaluated using (4) for the pairs of observations on uncertainties separately for each forecast horizon. As the natural and easily interpretable condition we set the bands of inflation in the US as 2%±1%, which is around the 2% of the US inflation target. The 2% target for inflation has been officially set in January 2012, but in practice was used earlier in the form of the ‘desired inflation’. In Canada, the 2% inflation target was established in late 1995. Figure 1 shows inflation in Canada and US with the ±1% bands indicated.

Figure 1

CPI Inflation in Canada and US, January 1992-October 2014



Usually, the term structure of inflation forecast is expressed by the sequence of standard deviations of uncertainty for each forecast horizon (see Clements, 2014). However, we have decided to express it by the average (across rolling windows) probabilities of the Canadian inflation being within the target bands. In the context of inflation targeting this seems to be a natural and more easily interpretable measure.

Let us denote by $\hat{f}_\tau(t) = f_{WSN_{\hat{\sigma}^{(\tau)}}}(t;\hat{\alpha}^{(\tau)},\hat{\beta}^{(\tau)},\hat{\sigma}^{(\tau)},-\hat{\sigma}^{(\tau)},0.75)$ the estimated WSN density function, where $\hat{\alpha}^{(\tau)},\hat{\beta}^{(\tau)},\hat{\sigma}^{(\tau)}$ are the SMD estimates of the WSN parameters for Canada ($\tau = 1$) and U.S. ($\tau = 2$). The corresponding *cdf*s are denoted by $\hat{F}_\tau$. The unconditional probabilities of the Canadian inflation being within the [a,b] range, where $a = 1$ and $b = 3$, are:

$$\int_a^b \hat{f}_1(x_1)dx_1 \ ,$$

The (conditional) probabilities of the Canadian inflation being within the [a,b] range, where $a = 1$ and $b = 3$, given that the US inflation is within the [1%,3%] are, following (3) and (5), given by:

$$\frac{\int_a^b\int_a^b c\left(\hat{F}_1(x_1),\hat{F}_2(x_2);\hat{\theta}\right)\hat{f}_1(x_1)\hat{f}_2(x_2)dx_1dx_2}{\int_a^b \hat{f}_2(x_2)dx_2} \ ,$$

where $\hat{\theta}$ is the estimated parameter of the Frank's copula.[6]

Table 2 gives, in columns (1) and (5), the averaged unconditional probabilities of inflation being within the [1%,3%] interval, and, in columns (2) and (6), conditional probabilities for selected forecast horizons, for the headline and core inflation in Canada, respectively. Standard errors are reported in brackets below the averages. In columns (3), (4), (7) and (8) the corresponding rudimentary sharpness measures of forecasts are given (see Gneiting, Balabdaoui and Raftery, 2007; and Mitchell and Wallis, 2011). The idea of sharpness measures is such that the density forecast should be concentrated around the realised value (observed *ex-post*) if the model forecasts accurately. The measure used here is the average (unconditional or conditional) probability of the Canadian inflation being within the [1%,3%] band computed for the cases where inflation (*ex-post* forecast realisation) actually was within this band. For a 'sharp' forecast, such measure should be higher than the corresponding unconditional and conditional probabilities.

Table 2

Average Probabilities of Inflation in Canada Being in the Interval [1%, 3%]

for. hor	headline inflation				core inflation			
	uncond. prob.	cond. prob.	sharpn. uncond. prob.	sharpn. cond. prob.	uncond. prob.	cond. prob.	sharpn. uncond prob.	sharpn. cond. prob.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
3	0.69 (0.32)	0.73 (0.33)	0.72 (0.30)	0.79 (0.30)	0.79 (0.40)	0.96 (0.16)	0.79 (0.39)	0.97 (0.14)
6	0.62 (0.26)	0.67 (0.27)	0.61 (0.27)	0.70 (0.27)	0.89 (0.29)	0.96 (0.15)	0.88 (0.30)	0.96 (0.15)
9	0.58 (0.22)	0.62 (0.22)	0.56 (0.23)	0.61 (0.23)	0.87 (0.31)	0.96 (0.16)	0.87 (0.33)	0.95 (0.16)
12	0.53 (0.20)	0.58 (0.19)	0.54 (0.19)	0.57 (0.19)	0.88 (0.28)	0.95 (0.16)	0.90 (0.28)	0.95 (0.17)
15	0.50 (0.17)	0.52 (0.17)	0.51 (0.18)	0.52 (0.18)	0.93 (0.21)	0.94 (0.17)	0.93 (0.20)	0.94 (0.17)
18	0.48 (0.16)	0.49 (0.17)	0.49 (0.15)	0.48 (0.16)	0.92 (0.21)	0.94 (0.18)	0.92 (0.21)	0.93 (0.18)
21	0.45 (0.15)	0.46 (0.15)	0.47 (0.14)	0.47 (0.13)	0.92 (0.20)	0.93 (0.17)	0.93 (0.20)	0.93 (0.17)
24	0.43 (0.14)	0.44 (0.14)	0.42 (0.15)	0.45 (0.13)	0.92 (0.20)	0.94 (0.15)	0.99 (0.20)	0.94 (0.15)

All probabilities in Table 2 decline monotonously with the increase in the forecast horizon, indicating a typical forecast term structure (or fan chart) pattern, where the uncertainty increases with the increase in the forecast horizon. The conditional probabilities are, as expected, higher than the corresponding unconditional ones. The differences diminish with the increase in the forecast horizon, indicating some convergence of the unconditional and

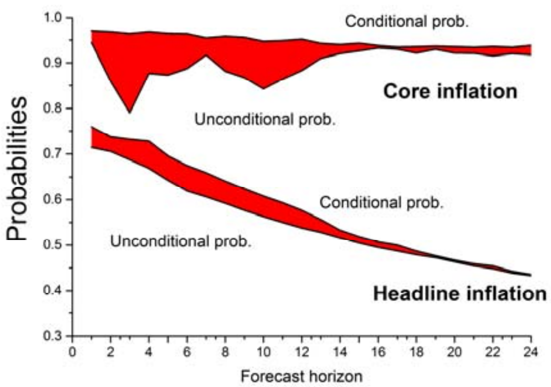
[6] *Programming has been made in GAUSS 12 and computations performed on the high powered parallel computer HPC ALICE at the University of Leicester. Computational details and codes are available from the authors.*

conditional distributions. This is also illustrated in Figure 1, where the probabilities are plotted for all forecast horizons up to 24. The sharpness measure is, in most cases greater than the corresponding probabilities. Standard deviations for all probabilities are relatively high, particularly for shorter forecast horizons. This might indicate changes in the parameters of the estimated distributions over time.

The probabilities obtained for core inflation uncertainty are markedly higher than that for the headline inflation uncertainty. Also, the differences in the conditional and unconditional probabilities are greater for core rather than headline inflation, particularly for shorter forecast horizons. On the one hand, it confirms that the core rather than headline inflation has been efficiently targeted. On the other hand, however, it indicates that the effects of the U.S. inflation uncertainty onto the Canadian core inflation uncertainty are greater than that on the headline. As the U.S. inflation is outside the reach of the Canadian monetary policy, it suggests a possible way of improvement in setting up an effective inflation indicator for Canada, which should be net of the U.S. inflation effects. High probabilities of hitting the band close to the inflation target, both unconditional and conditional, confirms the rationale of the way Bank of Canada constructs its core inflation measure.

Figure 2

Average Conditional and Unconditional Probabilities of Hitting the [1% - 3%] Inflation Band in Canada

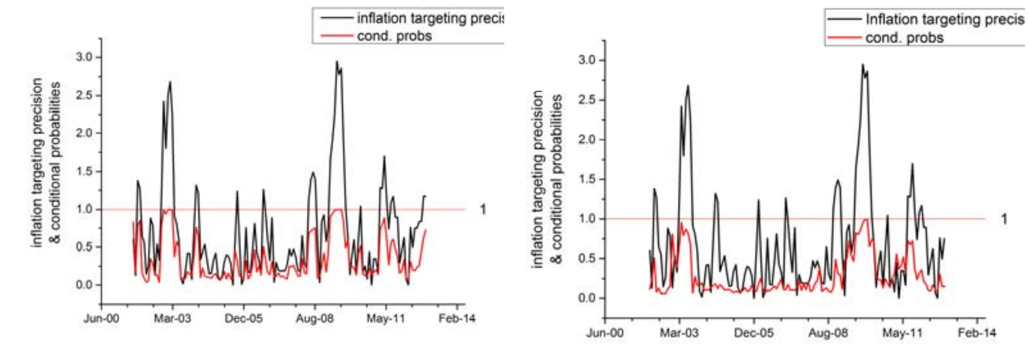


With the use of the conditional and unconditional probabilities of inflation being within the [a,b] range it is possible to forecast effectively the inflation targeting precision. Figure 3 shows plots of such conditional probabilities for Canada for forecast horizons 2 and 7 plotted alongside the simple measure of the precision of inflation targeting, defined as the absolute value of the difference between the headline inflation and target inflation (that is, 2%). The forecast has been shifted backwards by one horizon, so that the two-steps ahead probabilities are plotted against inflation observed in time $t+h-1$, that is, are treated as one-step ahead forecasts. Analogously, the seven-steps ahead probabilities are treated as six-steps ahead forecasts. For the sake of plot clarity, we have plotted the complements of the conditional probabilities to one, that is the probabilities that inflation is outside the [1%,3%,] range rather than inside.

Figure 3

Inflation Targeting Precision in Canada and the Conditional Probabilities of Inflation Being Outside the [1%,3%] Range, January 2003-December 2013

$h=2$	$h=7$



The plots show reasonable accuracy in explaining deviations of inflation from target, even of the reasonably large horizon. The probabilities that the short-term forecast has not missed the target band follow closely the inflation targeting precision, relevant large deviations and the longer-term forecast, with probabilities approaching unity for January-March 2003 and May-September 2008. The main peaks in inflation targeting precision for the longer-term forecast also coincide with the conditional probabilities approaching unity.

On the basis of the probabilities discussed above, we can construct an uncertainty measure that, unlike the squares of the ARMA-GARCH forecast errors (see Table 1) correlates with the EPU index. It can be formulated as a squared forecast error scaled by the odds of the U.S. inflation being outside the [1%,3%] zone, by the unconditional probabilities. Denoting such measure by $um_{t,h}$ we can write is as

$$um_{t,h} = \left(U_{t,h}^{(1)} \times \frac{1 - \int_a^b \hat{f}_2(x_2)dx_2}{\int_a^b \hat{f}_2(x_2)dx_2} \right)^2 ,$$

where: $a=1$ and $b=3$

and, as before, Canada and U.S are denoted by 1 and 2, respectively. The intuition here is such that and increased odds for uncertainty in the U.S. being outside the range affects positively the Canadian uncertainty. As such information is might find its way to the media (but not to the VAR-BEKK-GARCH model directly), such correction should increase the correlation of the new uncertainty measure with the EPU index. Table 3 gives the Spearman's rank correlation measures of $um_{t,h}$ with EPU for $h=1,2,...,12$.

Table 3

Spearman's Rank Correlation Coefficients between EPU and $um_{t,h}$

f.hor	1	2	3	4	5	6
	0.09	**0.11**	0.22	0.39	0.31	0.36
f.hor	7	8	9	10	11	12
	0.41	0.42	0.38	0.42	0.37	0.37

Legend: coefficients not significant at the 10% significance level are boldfaced. P-values used for testing have been obtained by simple bootstrap.

The rank correlation coefficients for forecast horizons of 1 and 2 remain insignificant, as they are for some other countries listed in Table 1. However, for longer forecast horizons, the coefficients become significant, which is in line with the results of the correlation of EPU with inflation forecast uncertainty for other countries.

6. Conclusions

We managed to shed new light on the puzzling absence of correlation of the inflation forecast uncertainty and the economic policy uncertainty index. We argue that the presence of dependence between such uncertainties between countries might cause such effect. For such cases, we propose a new method for constructing an inflation term structure. The method is conceptually simple, albeit computationally awkward. Its application can lead to an improvement in foreseeing uncertainty related to inflation and enables computation of term structure relatively to the performance of another country, or economic alliance. It also suggests a potentially new way of computing uncertainty measures. We exemplify the concept by the analysis of the Canadian inflation forecast term structure, but our technique can also be applied, for instance, for evaluating the inflation forecast term structure for the European Union countries outside the Eurozone relatively to the policy of the European Central Bank. For Canada, the results look promising. It has been possible to forecast effectively the deviations of inflation from its target using conditional and unconditional *ex-ante* probabilities of inflation being within a certain band around the target. The results also confirm the rationale for using core inflation in inflation targeting and suggest a way of eliminating the effect of external inflation uncertainty onto such a measure.

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